

SANS DATA ANALYSIS

Boualem Hammouda

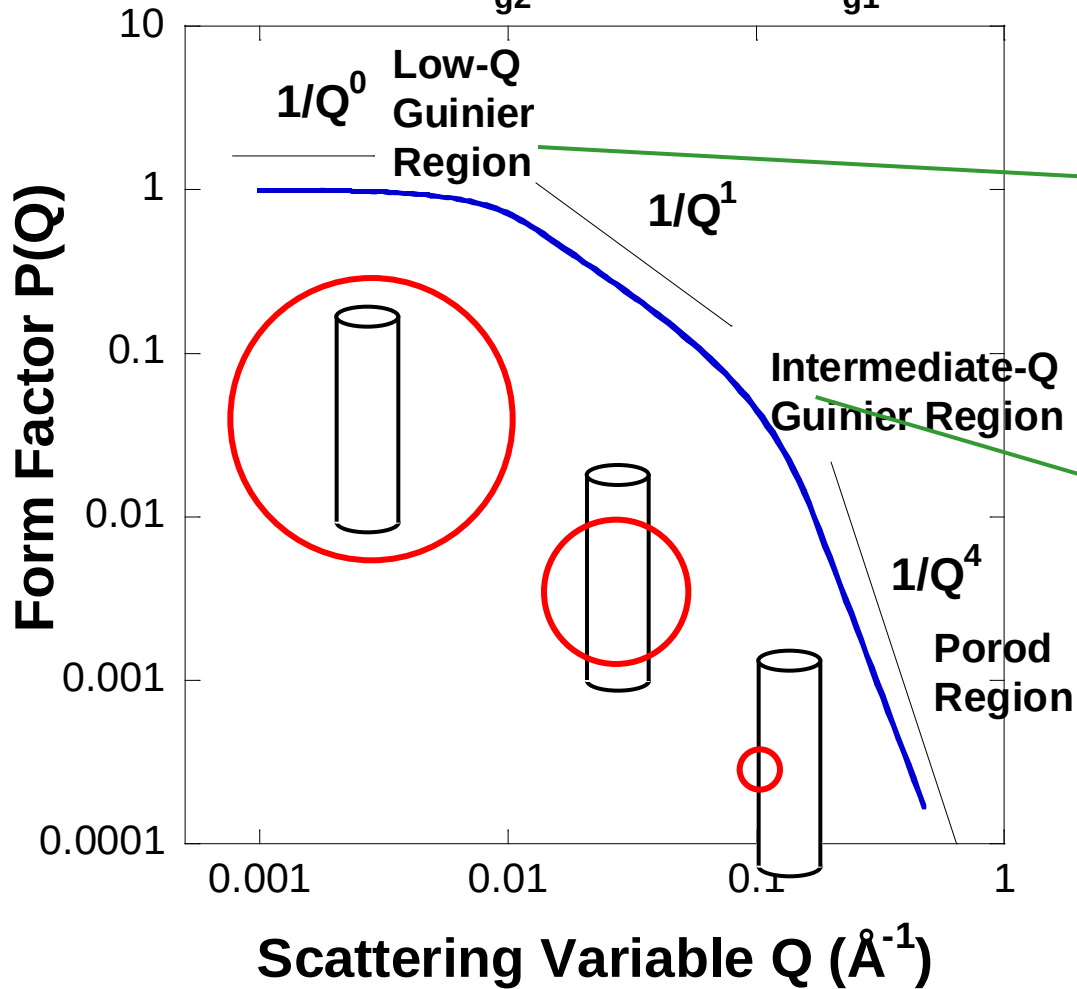
National Institute of Standards and Technology
Center for Neutron Research

- 1. Standard Plots (Guinier Plot, Porod Plot)
- 2. SANS Models
- 3. Inverse Fourier Transform
- 4. Shape Reconstruction Method

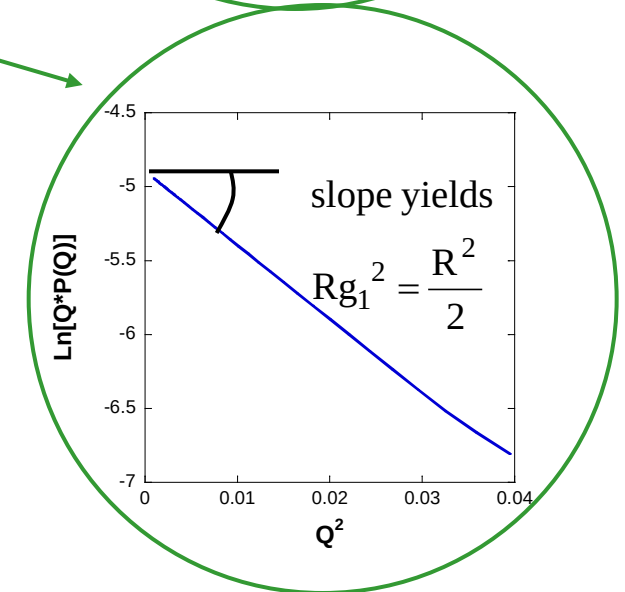
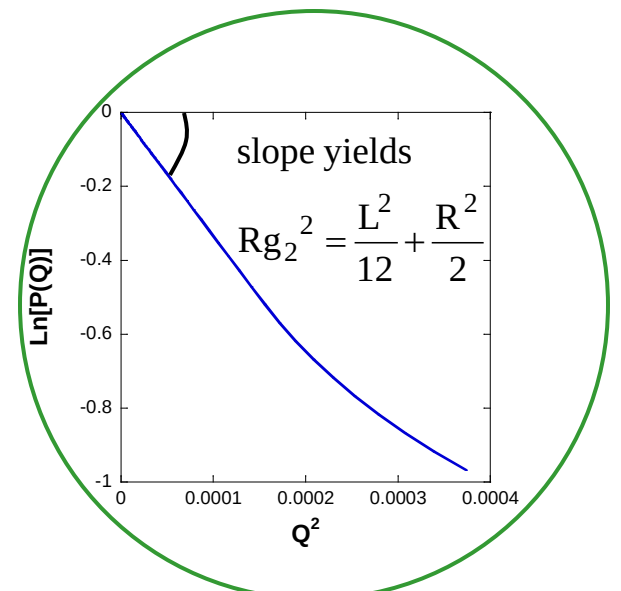
1. Standard Plots

Guinier-Porod Regions

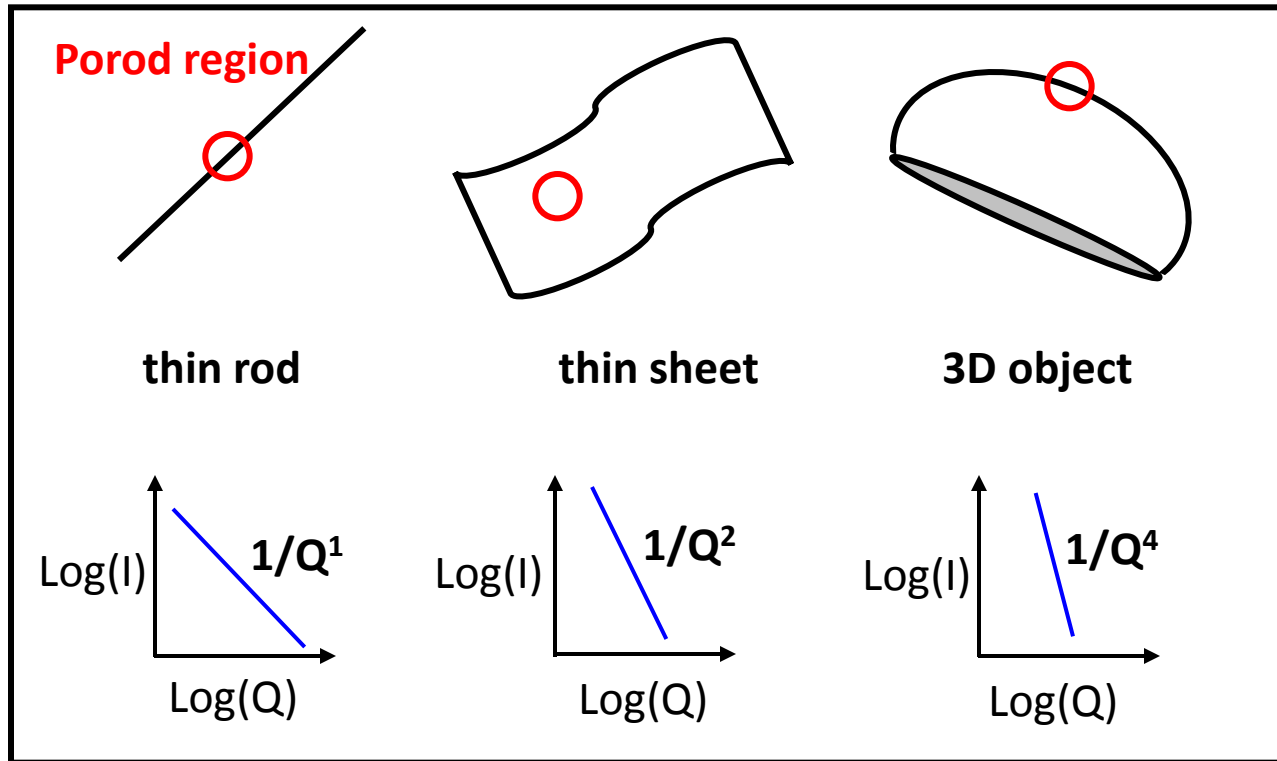
Cylinder with $R_{g2} = 100 \text{ \AA}$ and $R_{g1} = 10 \text{ \AA}$



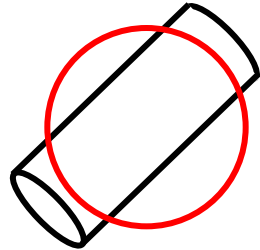
Guinier Plots



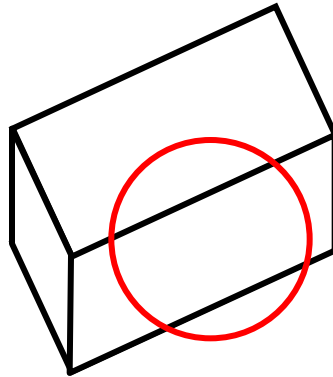
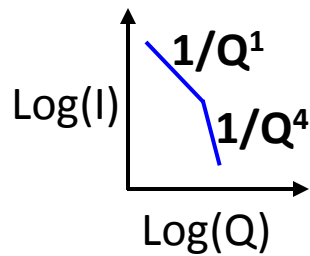
Assortment of Porod Exponents



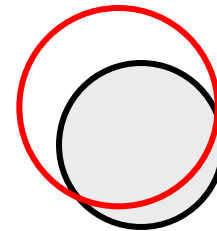
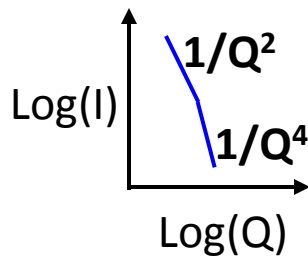
More Porod Exponents



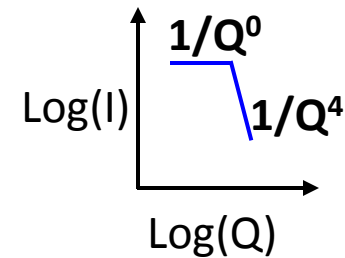
thick cylinder



thick slab

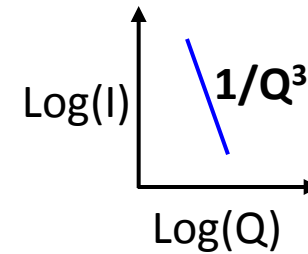
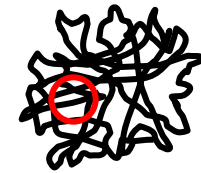
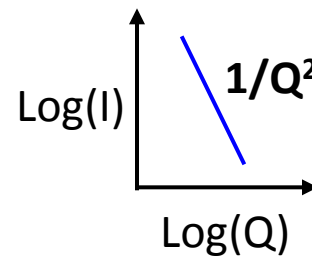
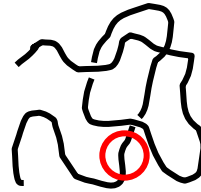
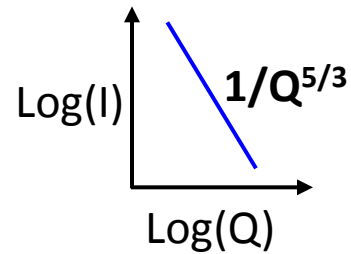
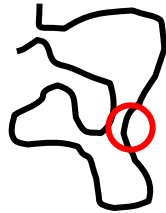


3D object

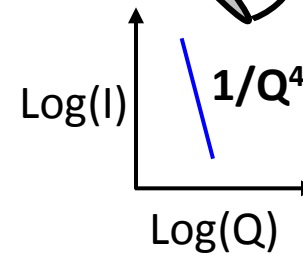
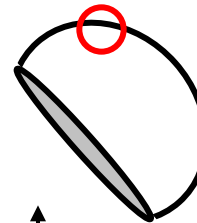
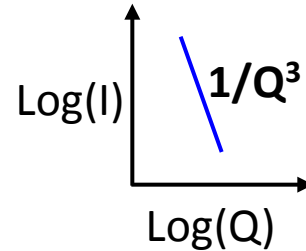
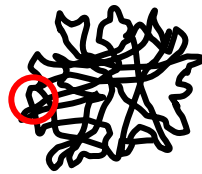


More Porod Exponents

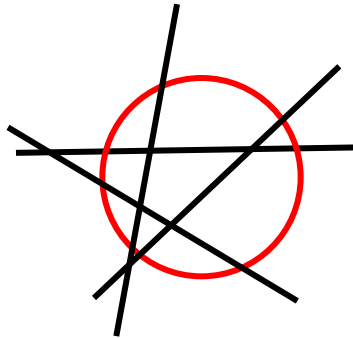
MASS
FRACTALS



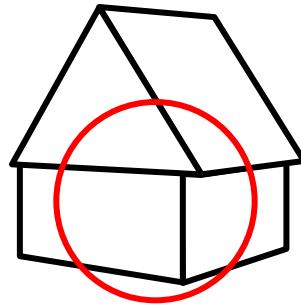
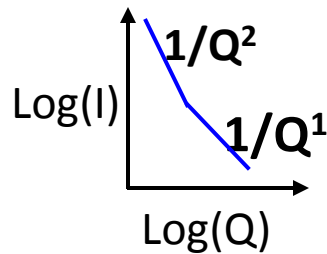
SURFACE
FRACTALS



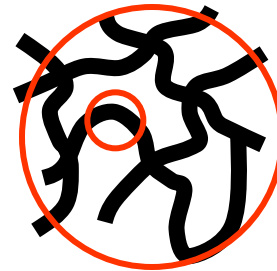
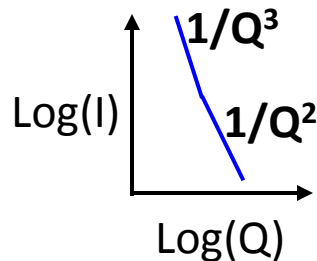
More Porod Exponents



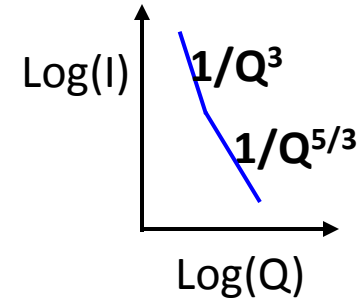
Rod network



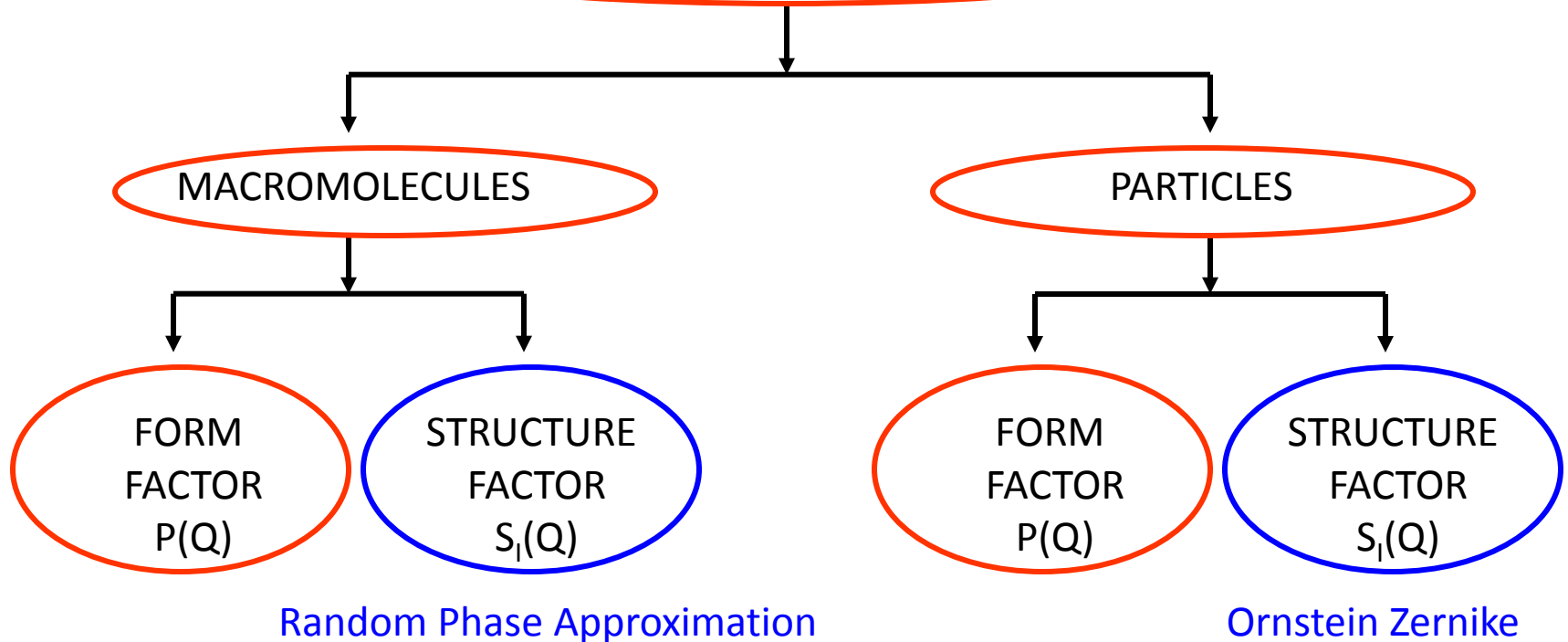
Sheet network



Another structure



2. SANS Models

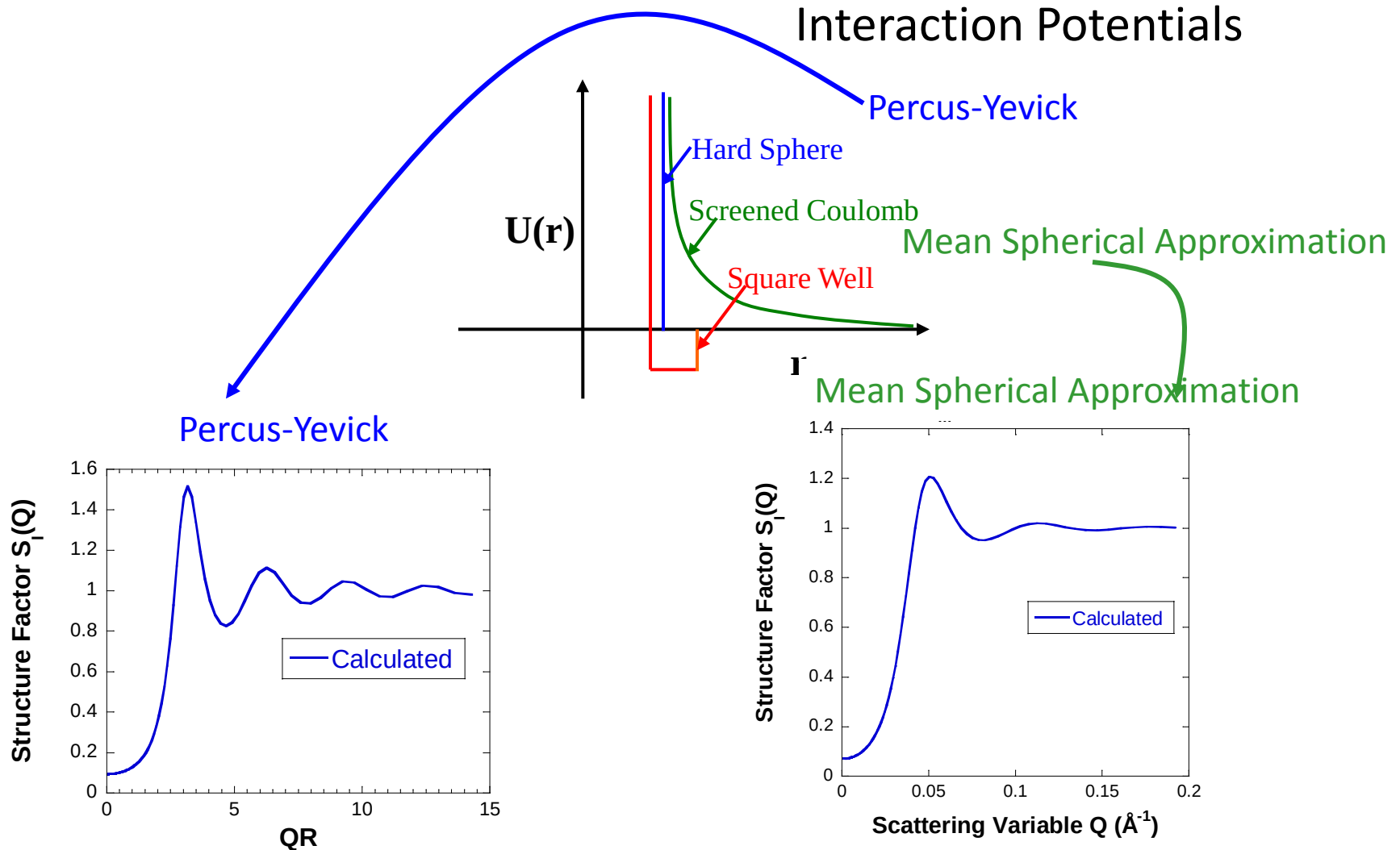


$$\frac{d\Sigma(Q)}{d\Omega} = \phi_A (\rho_A - \rho_B)^2 V_A P(Q) S_I(Q)$$

Diagram illustrating the components of the SANS equation:

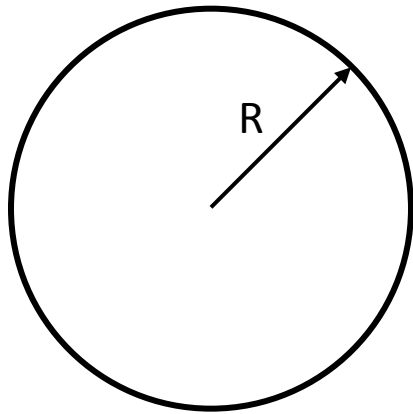
- $\frac{d\Sigma(Q)}{d\Omega}$: cross section
- ϕ_A : volume fraction
- $\rho_A - \rho_B$: contrast factor
- V_A : particle volume
- $P(Q)$: form factor
- $S_I(Q)$: structure factor

The Ornstein-Zernike Equation

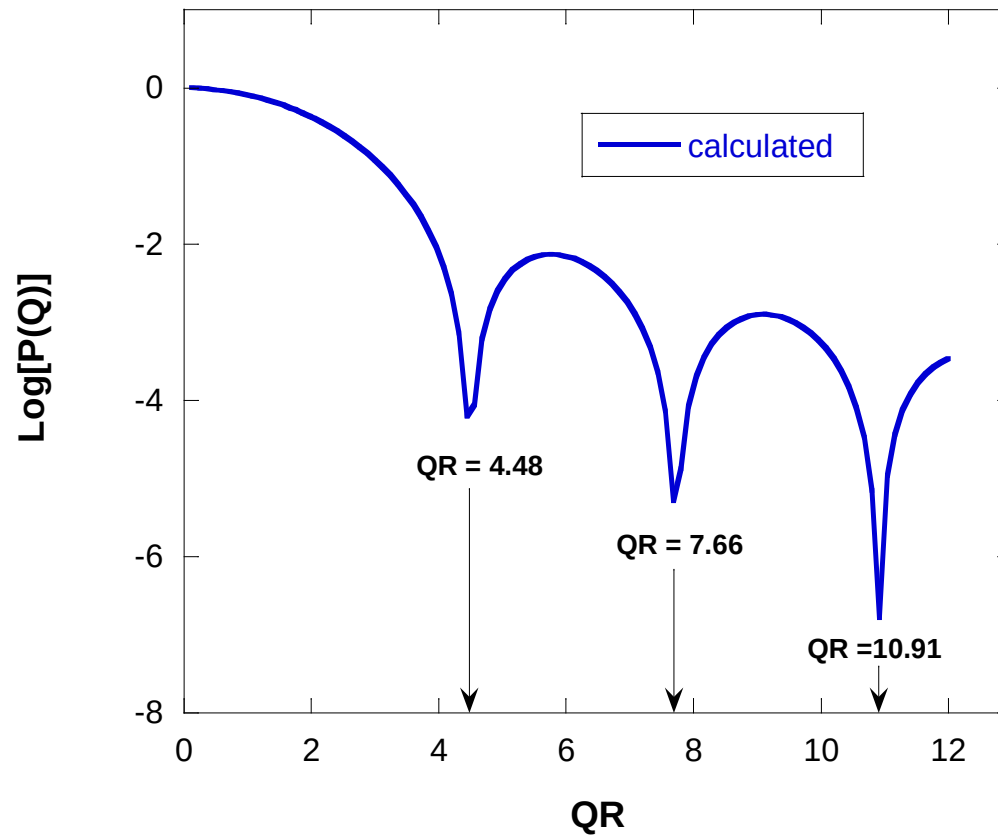


FORM FACTOR FOR A SPHERE

$$P(Q) = \left[\frac{3j_1(QR)}{QR} \right]^2 = \left[\frac{3}{QR} \left(\frac{\sin(QR)}{(QR)^2} - \frac{\cos(QR)}{QR} \right) \right]^2$$

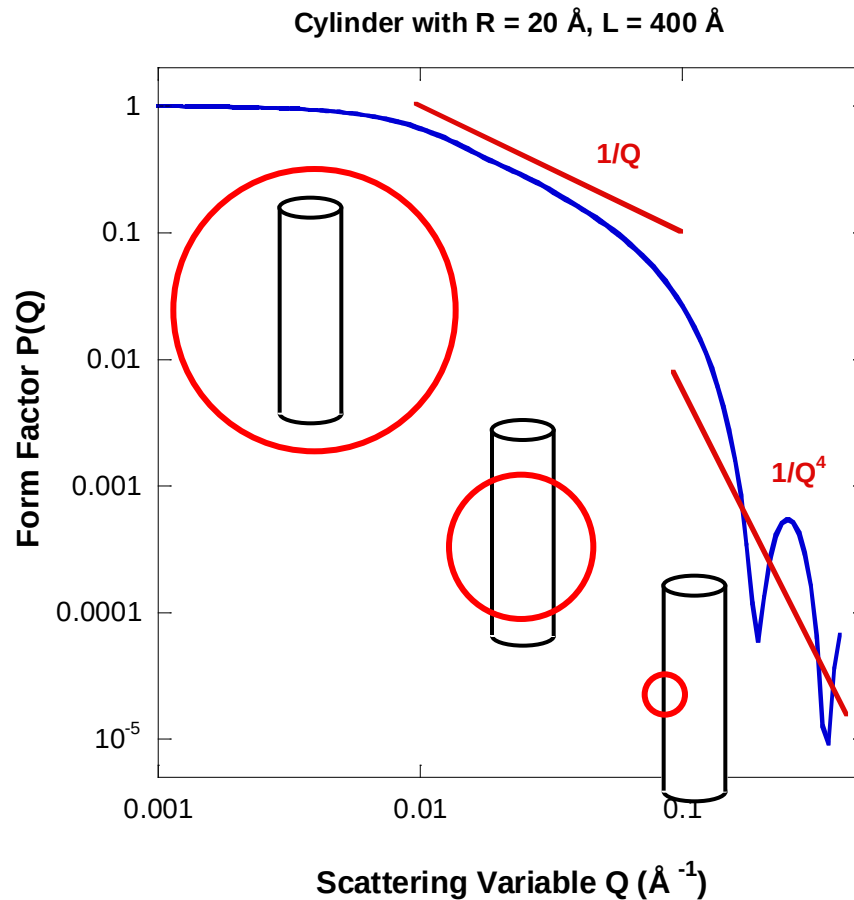
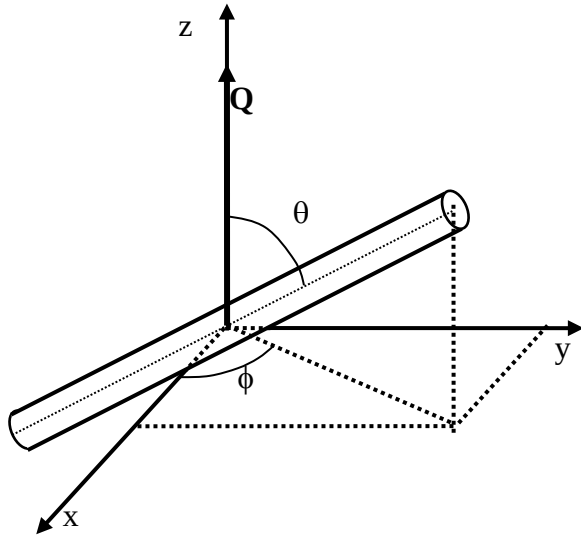


Form Factor for a Sphere

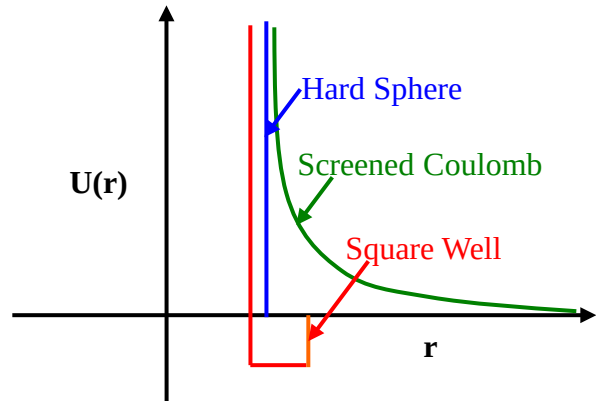


FORM FACTOR FOR A CYLINDER

$$P(Q) = \frac{1}{2} \int_{-1}^1 d\mu \left[\frac{\sin(Q\mu L / 2)}{Q\mu L / 2} \right]^2 \left[\frac{2J_1(Q\sqrt{1-\mu^2} R)}{Q\sqrt{1-\mu^2} R} \right]^2$$

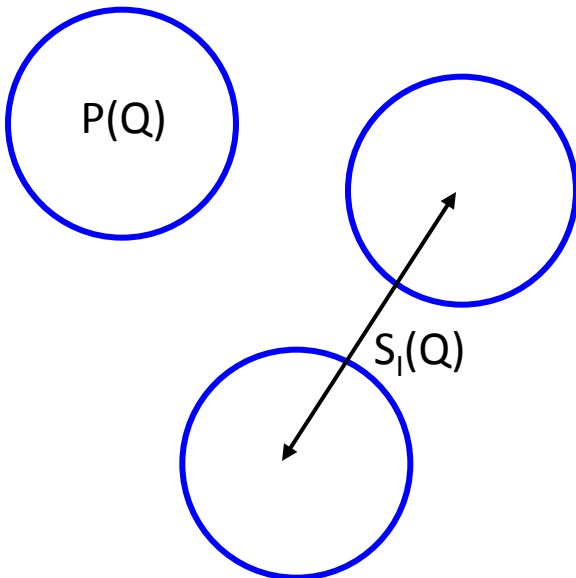
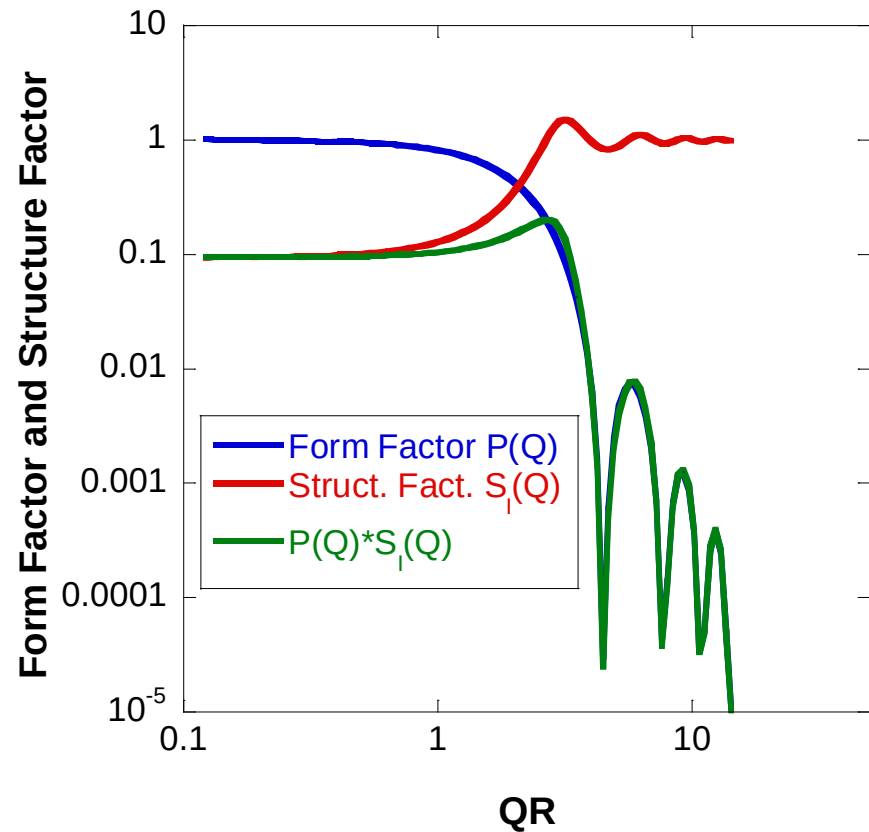


STRUCTURE FACTOR – THE ORNSTEIN-ZERNIKE EQUATION



$$\frac{d\Sigma(Q)}{d\Omega} = \phi_A (\rho_A - \rho_B)^2 V_A P(Q) S_I(Q)$$

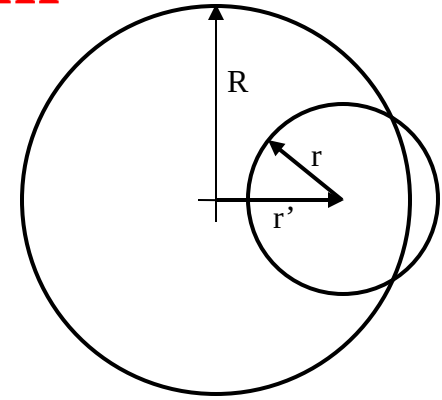
Percus-Yevick Model



3. Fourier Transform

Density-density correlation function:

$$P(Q) = \frac{\langle n(-Q)n(Q) \rangle}{n^2} = \int d\vec{r} \int d\vec{r}' \frac{\langle n(r)n(r') \rangle}{n^2} \exp[i\vec{Q} \cdot (\vec{r}' - \vec{r})]$$

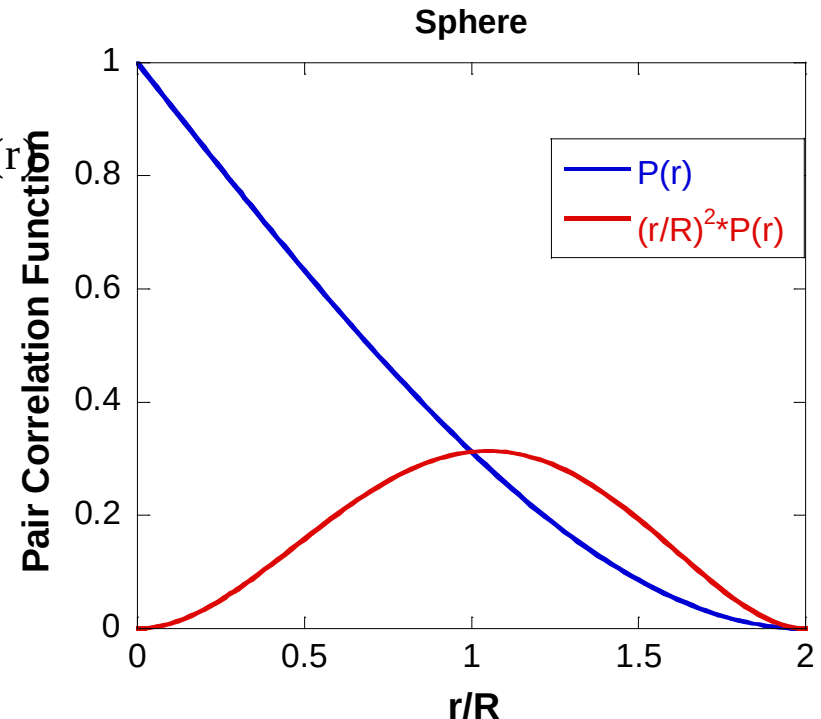


Fourier transform:

$$P(Q) = \int d^3r \exp[-i\vec{Q} \cdot \vec{r}] P(\vec{r}) = \frac{1}{V_P} \int_0^\infty dr 4\pi r^2 \frac{\sin(Qr)}{Qr} P(r)$$

Radial pair correlation function:

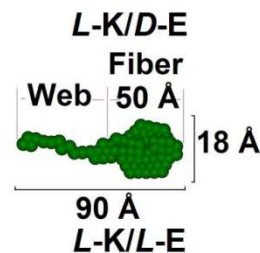
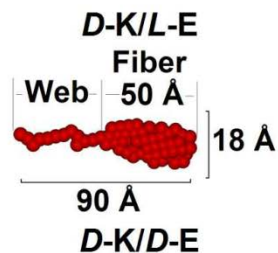
$$P(r) = 1 - \frac{3}{4} \left(\frac{r}{R} \right) + \frac{1}{16} \left(\frac{r}{R} \right)^3$$



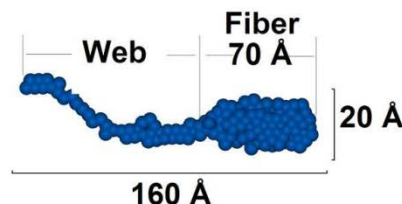
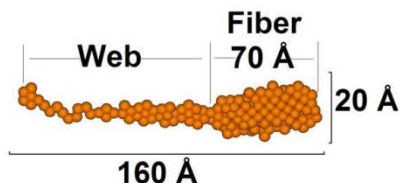
SANS Data Analysis

A

Heterochiral



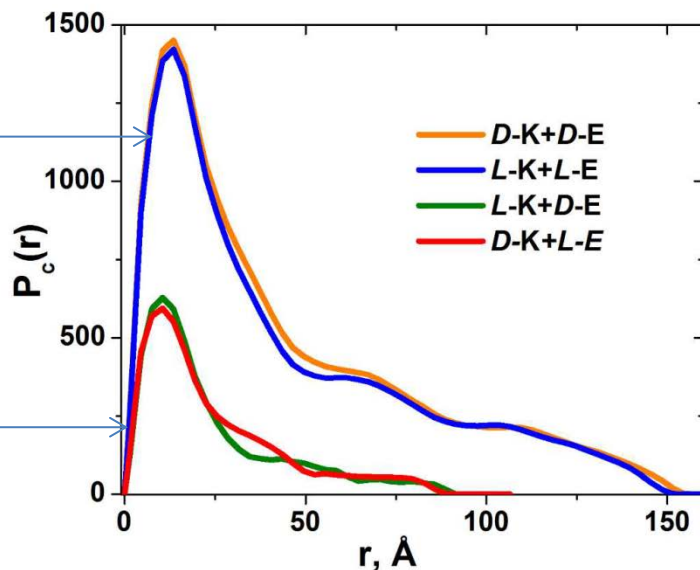
Homochiral



Shape Reconstruction
Fiber cross sections

B

Homochiral



Heterochiral

Inverse Fourier
Transform

- Homochiral fibers are thicker and denser than heterochiral ones

SANS DATA ANALYSIS METHODS

- 1. Standard Plots (Guinier Plot, Porod Plot)
- 2. SANS Models
- Igor Pro
- SASVIEW